Energy Conversion
sheet 1

(1) $N=300$ $I=1A$ $\mu_r=300$ $L_c=60\,cm$ $l_g=1mm$

$$NI = \phi R = \phi [R_c + R_g]$$

$$NI = \phi \left[\frac{L_c}{\mu_0 \mu_r A_c} + \frac{L_g}{\mu_0 A_g} \right] \qquad A_c = A_g$$

$$NI = B \left[\frac{L_c}{\mu_0 \mu_r} + \frac{L_g}{\mu_0} \right]$$

$$B = \frac{300 \times 1}{\left[\frac{0.6}{300\mu_0} + \frac{10^{-3}}{\mu_0} \right]} = 0.125\ T$$

---

(2) $L=1m$ $A=0.001\,m^2$ $l_g=5mm$ $\phi = 1\times10^{-3}\,wb$ $N=200$

$\mu_r=500$

$$NI = \phi [R_c + R_g] = \phi \left[\frac{L_c}{\mu_0 \mu_r A} + \frac{L_g}{\mu_0 A} \right]$$

$$200\,I = \left[\frac{1}{500\,\mu_0 (0.001)} + \frac{5*10^{-3}}{0.001\,\mu_0} \right] \times 10^{-3}$$

$$I = 27.85A$$

Scanned by CamScanner

(3) $A = 15\,cm^2$ $l_c = 70\,cm$ $l_g = 2.5\,mm$ $\mu_r = 1500$ $I = 1A$

$$R_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.7}{1500 \mu_0 \times 15 \times 10^{-4}} = 247574.35 \text{ AT/wb}$$

$$R_g = \frac{l_g}{\mu_0 A_g} = \frac{2.5 \times 10^{-3}}{15 \mu_0 \times 10^{-4}} = 1.326 \times 10^6 \text{ AT/wb}$$

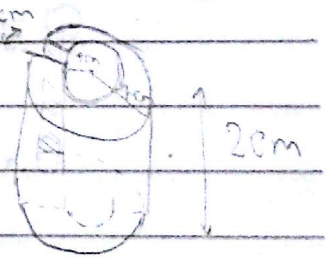
$$R_t = 1.574 \times 10^6 \text{ AT/wb}$$

$$\boxed{\phi = \frac{NI}{R_t} = 4.76 \times 10^{-5} \text{ wb}}$$

$$\lambda = N\phi = 3.57 \times 10^{-3} \text{ wb-turn}$$

$$L = 3.57 \text{ mH}$$

(4) $N = 75$ $\mu_r = 750$ $l_g = 0.5\,cm$



$$l_c = 2\pi(3.5) - 0.5 = 21.5\,cm$$
$$A_c = A_g = 2\,cm^2$$

$$R_c = \frac{l_c}{\mu_0 \mu_r A_c} = 1140610.4 \text{ AT/wb}$$

$$R_g = \frac{l_g}{\mu_0 A} = 19894367.9 \text{ AT/wb}$$

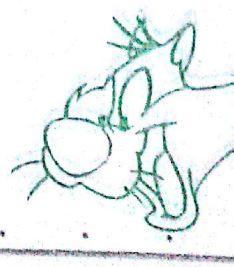
$R_t = 21.053 \times 10^6$

$$L = \frac{N^2}{R_t} = \frac{(75)^2}{R_c + R_g} = 267.4 \text{ MH}$$

$$\phi = B_g A_g = 1.2 \times 2 \times 10^{-4} = 0.24 \text{ mwb}$$

$$I = \frac{\phi R}{N} = 67.3 \text{ A}$$

$$\lambda = N\phi = 0.018 \text{ wb-turn}$$

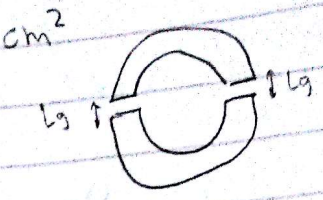


Scanned by CamScanner

(5) $\mu_s = 825$ $\mu_i = 165$ $l_g = 0.2\,mm$ $A = 10\,cm^2$

$L = 10.5\,\pi\,cm$ (for each section)

$l_s = l_i = 33\,cm$



$$mmf = NI = \phi(R_s + R_i + 2R_g)$$

$$= \phi\left[\frac{l_s}{\mu_s \mu_0 A} + \frac{l_i}{\mu_i \mu_0 A} + \frac{2l_g}{\mu_0 A}\right]$$

$$= \frac{5 * 10^{-4}}{10 \times 10^{-4}}\left[\frac{0.33}{825\mu_0} + \frac{0.33}{165\mu_0} + \frac{2 * 0.2 \times 10^{-3}}{\mu_0}\right]$$

$$mmf = 1114.1\ A.T$$

---

(6) $L_a = 8\,cm$ $A_a = 0.3\,cm^2$ $N = 4000$

$L_b = 6\,cm$ $A_b = 0.9\,cm^2$ $B_g = 0.3\,wb/m^2$

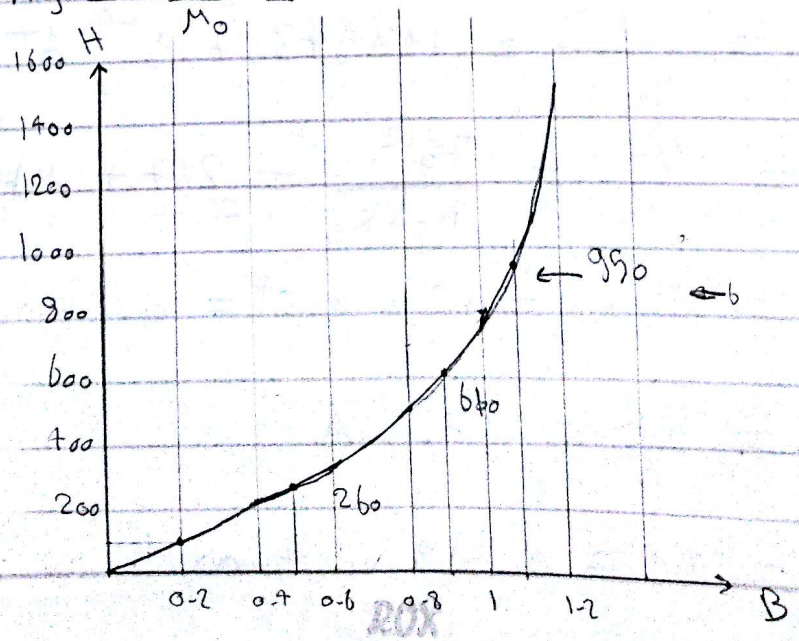
$l_g = 0.5\,mm$ $A_g = 1.5\,cm^2$

$$\phi = B_g A_g = 4.5 \times 10^{-5}\ wb$$

$$B_a = \frac{\phi}{A_a} = 0.9\,T$$

$$B_b = \frac{\phi}{A_b} = 0.5\,T$$

$$H_g = \frac{B_g}{\mu_0} = 238732.4\ AT/m$$



Scanned by CamScanner

From B-H curve:

$H_a = 610$ AT/m $\quad H_b = 260$ AT/m

$NI = H_a l_a + H_b l_b + H_g l_g$

$4000\, I = 610 \times 8\times10^{-2} + 260 \times 6\times10^{-2} + 238732 \times 0.5\times10^{-3}$

$\boxed{I = 45.94 \text{ mA}}$

(7) $N = 500$ $\quad l_c = 360$ mm $\quad l_g = 1.5$ mm $\quad B = 0.8$T

$H_c = 510$ AT/m $\quad H_g = \frac{B}{\mu_0} = 636619.7$ AT/m

$NI = H_c l_c + 2 H_g l_g$

$500\, I = 510 \times 360 \times 10^{-3} + 2 \times 636619.7 \times 1.5 \times 10^{-3}$

$\boxed{I = 4.187 \text{ A}}$

$B_c = \mu_0 \mu_r H_c$

$0.8 = 510\, \mu_0 \mu_r$

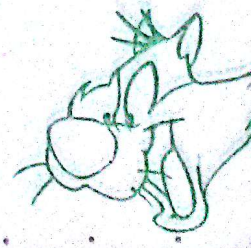
$\mu = \mu_0 \mu_r = 1.568 \times 10^{-3}$ $\frac{\text{Tesla.m}}{\text{A.T}}$

$\mu_r = 1248.2$

(e) $NI = H_c l_c$

$I = \frac{510 \times 360 \times 10^{-3}}{500}$

$\boxed{I = 0.3672 \text{ A}}$



Scanned by CamScanner

(8)

$$NI = \Phi R$$
$$= \Phi [R_c + 2R_g]$$

$$NI = \frac{\Phi}{A}\left[\frac{L_c}{\mu_c} + 2\frac{L_g}{\mu_0}\right]$$

$$\mu_c = \frac{B_c}{H_c}$$

$$4 \times 500 = B\left[\frac{L_c H_c}{B} + 2\frac{L_g}{\mu_0}\right]$$

$$\boxed{2000 = 0.36\, H_c + 1591.5\, B}$$

intersection between this line and B-H curve gives the operating point

Scanned by CamScanner

(9) $l_g = 1mm \quad l_c = 0.2m \quad A_c = 0.001 m^2 \quad N = 100$

(a) $\phi = 1.06 mwb$

$$B = \frac{\phi}{A} = 1.06 T$$

$$H_g = \frac{B}{\mu_0} = 843521.2 \ AT/m$$

$H_c = 1000 \ AT/m$ (from B-H curve)

$$NI = H_c l_c + H_g l_g$$

$$100 I = 1000 \times 0.2 + 843521.2 \times 10^{-3}$$

$$\boxed{I = 10.43 \ A}$$

$$\phi R_c = H_c l_c$$

reluctance of cast steel

$$R_c = \frac{1000 \times 0.2}{1.06 \times 10^{-3}} = 188679.2 \ AT/wb$$

$$R_g = \frac{l_g}{\mu_0 A} = \frac{10^{-3}}{0.001 \mu_0} = 795774.7 \ AT/wb$$

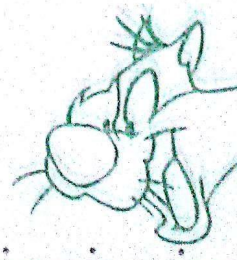
(b) For cold rolled steel :-

$$H_c = 18 \ AT/m$$

$$100 I = 18 \times 0.2 + 843521.2 \times 10^{-3}$$

$$\boxed{I = 8.47 \ A}$$

$$R_c = \frac{18 \times 0.2}{1.06 \times 10^{-3}} = 3396.2 \ AT/wb$$

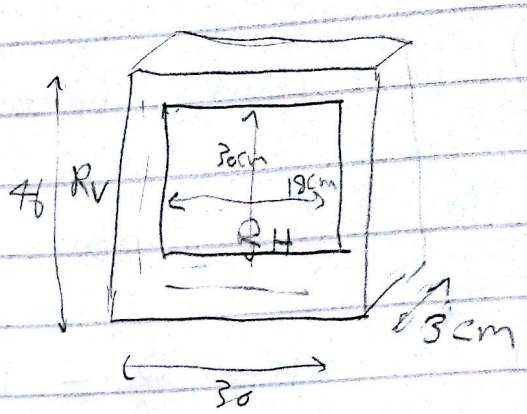


Scanned by CamScanner

Date: No:

(10)

$$mmf = 2H_V L_V + 2H_H L_H$$



$$L_V = \frac{46+30}{2} = 38\ cm$$

$$L_H = \frac{30+18}{2} = 24\ cm$$

$$A_V = (30-18)3 = 36\ cm^2$$
$$A_H = (46-30)3 = 48\ cm^2$$

$$B_V = \frac{\Phi}{A_V} = \frac{4.8 * 10^{-3}}{36 * 10^{-4}} = 1.33\ T$$

$$B_H = \frac{\Phi}{A_H} = 1\ T$$

From B-H curve:

$$H_V = 38\ AT/m \qquad H_H = 18\ AT/m$$

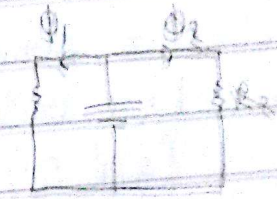
$$mmf = 2 \times 38 \times 0.38 + 2 \times 18 \times 0.24$$

$$\boxed{mmf = 37.52\ AT}$$

Scanned by CamScanner

Date:

(11) $\phi_1 = 2.515 \times 10^{-2}$ wb



$$R_1 = \frac{l_{g1}}{\mu_0 A} = \frac{0.5 \times 10^{-3}}{\mu_0 \times 10 \times 10^{-4}} = 397887.4 \text{ AT/wb}$$

$$R_2 = \frac{l_{g2}}{\mu_0 A} = 2R_1 = 795774.7 \text{ AT/wb}$$

$$NI = \phi_1 R_1$$

$$I = \frac{2.515 \times 10^{-2} * 397887.4}{250} = 40 \text{ A}$$

$$\boxed{V = IR = 120 \text{ V}}$$

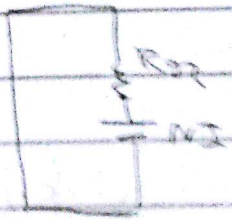
$$NI = \phi_2 R_2$$

$$\phi_2 = \frac{40 \times 250}{795774.7} = \boxed{12.566 \times 10^{-3} \text{ wb}}$$

---

(12)

$$I = \frac{V}{R} = \frac{120}{3} = 40 \text{ A}$$



$$R_{g2} = 795774.7 \text{ AT/W}$$

$$\phi_2 = 12.566 \times 10^{-3} \text{ wb}$$

$$\phi_1 = 0$$

ϕ_2 is the same as problem (11) because N, I, R hasn't changed

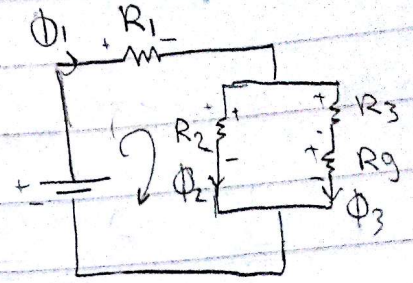
Scanned by CamScanner

(13) $L_1 = 0.8\,m$ $A_1 = 0.05\,m^2$

$L_2 = 0.4\,m$ $A_2 = 0.03\,m^2$

$L_3 = 0.3\,m$ $A_3 = 0.0286\,m^2$

$L_g = ??$ $A_g = 0.0286\,m^2$



$$\Phi_1 = \Phi_2 + \Phi_3$$

$$\phi_1 = 4.2 * 10^{-2} + 2.8 * 10^{-2} = 7 * 10^{-2}\ wb$$

$B_1 = 1.4\,T \rightarrow H_1 = 2500\ AT/m$

$B_2 = 1.4\,T \rightarrow H_2 = 2500\ AT/m$

$B_3 = 1.05\,T \rightarrow H_3 = 1000\ AT/m$

→ KVL on loop abcfa

$$NI = H_1 L_1 + H_2 L_2$$
$$= 2500 \times 0.8 + 2500 \times 0.4$$

$$\boxed{NI = 3000\ AT}$$

→ KVL on loop adecba

$$H_2 L_2 = H_3 L_3 + H_g L_g$$

$$2500 \times 0.4 = 1000 \times 0.3 + \frac{1.05}{\mu_0} L_g$$

$$\boxed{L_g = 8.377 \times 10^{-4}\ m}$$

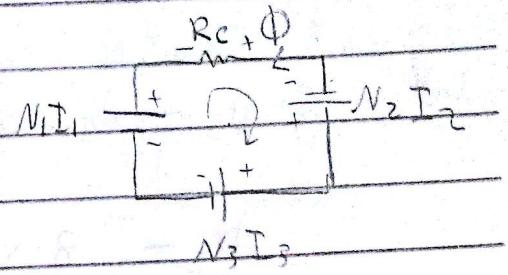
Scanned by CamScanner

(14) $A = 5 \times 10^{-4}$ $L = 0.4\,m$ $\Phi = 0.45\,mWb$

$N_A = 200$ $I_A = 0.5A$

$N_B = 400$ $I_B = 0.75A$

$N_C = 100$



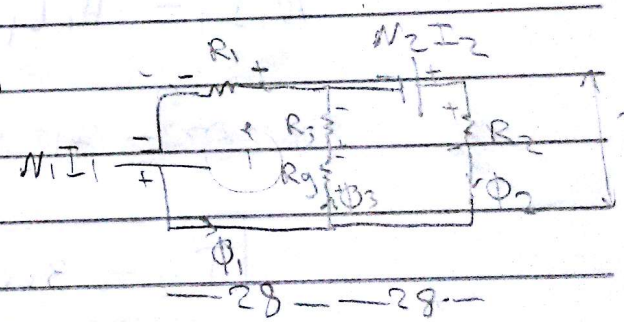
$$B_c = \frac{\Phi}{A} = 0.9\,T$$

$$H_c = 800\ AT/m$$

$$N_3 I_3 = N_1 I_1 + N_2 I_2 + H_c l_c$$

$$\boxed{I_3 = 7.2\,A}$$ from (a) to (b)

(15) $A = 50 \times 10^{-4}\,m^2$ $\Phi_3 = 5 \times 10^{-3}\,wb$



$$B_3 = \frac{5 \times 10^{-3}}{50 \times 10^{-4}} = 1\,T$$

$$H_3 = 18\ AT/m$$

$$H_g = \frac{B_3}{\mu_0} = 795774.7\ AT/m$$

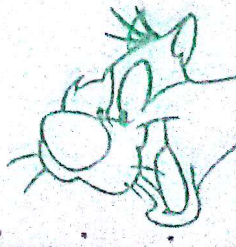
$L_1 = 79\,cm$ $L_2 = 79\,cm$ $L_3 = 23\,cm$ $l_g = 1\,mm$

KVL on loop 1:

$$N_1 I_1 = H_1 L_1 + H_3 l_3 + H_g l_g$$

$$3 \times 263 = 0.79 H_1 + 0.23 \times 18 + 10^{-3} \times H_g$$

$$\boxed{H_1 = -13.82\ AT/m}$$



Scanned by CamScanner

$B_1 = 0.6\ T$ (from B-H curve)

$\phi_1 = B_1 A = -3 \times 10^{-3}$ wb

$\phi_3 = \phi_1 + \phi_2$

$\phi_2 = 8 \times 10^{-3}$ wb

$B_2 = 1.6\ T$

$H_2 = 150$ AT/wb

KVL on the outer loop

$N_1 I_1 = H_1 L_1 + N_2 I_2 - H_2 L_2$

$3 \times 263 = -13.82 \times 0.79 + 300 I_2 - 150 \times 0.79$

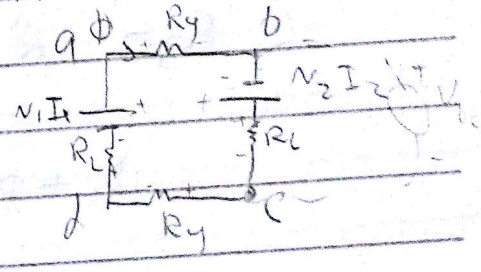
$\boxed{I_2 = 3.06\ A}$

from a to b

Scanned by CamScanner

16 $L_L = 8\,cm$ $A_L = 30\,cm^2$ $N_1 = 400$ $N_2 = 200$

$L_y = 10\,cm$ $A_y = 40\,cm^2$ $\phi = 4.8\,mWb$



(a) $B_L = \frac{\phi}{A} = 1.6\,T$

$B_y = 1.2\,T$

$H_L = 2000\ AT/m$ $H_y = 300\ AT/m$

$N_1 I_1 + N_2 I_2 = 2 H_L L_L + 2 H_y L_y$

($N_1 I_1 \to 0$)

$200\, I_2 = 2 \times 2000 \times 0.08 + 2 \times 300 \times 0.1$

$\boxed{I_2 = 1.9\ A}$

$mmf_{bc} = H_L L_L - N_2 I_2 = -220\ AT$

(b) $N_1 I_1 = 2 H_L L_L + 2 H_y L_y$

$\boxed{I_1 = 0.95\ A}$

$mmf_{bc} = H_L L_L = 160\ AT$

(c) $N_1 I_1 = H_L L_L$

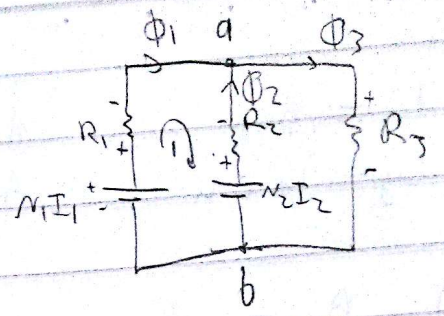
$\boxed{I_1 = 0.4\ A}$

$N_1 I_1 + N_2 I_2 = 2 L_L H_L + 2 H_y L_y$

$\boxed{I_2 = 1.1\ A}$

$\boxed{mmf_{bc} = -60\ AT}$

Scanned by CamScanner



(17) let: $R_2 = R'$

$R_1 = 3R' = 3R_2$

$R_3 = 3R' = 3R_2 = R_1$

$$\Phi_1 + \Phi_2 = \Phi_3$$

KVL on loop 1

$$N_1I_1 - N_2I_2 = \Phi_1 R_1 - \Phi_2 R_2$$

$$N_1I_1 - N_2I_2 = R_2[3\Phi_1 - \Phi_2] \longrightarrow (1)$$

KVL on the outer loop:

$$N_1I_1 = \Phi_1 R_1 + \Phi_3 R_3 = 3R_2[\Phi_1 + \Phi_3] \longrightarrow (2)$$

(a) $\Phi_1 = 0 \qquad \Phi_2 = \Phi_3$

$$100 \times 10 - 200 I_2 = -\Phi_2 R_2$$

$$100 \times 10 = 3\Phi_2 R_2$$

$$\Phi_2 R_2 = \frac{1000}{3}$$

$$200 I_2 = 1000 + \frac{1000}{3}$$

$$\boxed{I_2 = 6.67 \text{ A}}$$

(b) $\boxed{mmf_{ba} = -1000 \text{ AT}}$

Scanned by CamScanner

(c) $\phi_3 = 0$ $\phi_1 = -\phi_2$

$$N_1 I_1 - N_2 I_2 = \mathcal{R}\phi_1 R_2$$

$$\frac{1}{3} N_1 I_1 = \mathcal{R}\phi_1 R_2$$

$$N_1 I_1 - N_2 I_2 = \frac{4}{3} N_1 I_1$$

$$I_2 = -1.667 \text{ A}$$

Scanned by CamScanner

Date:

P-19

$P_e = K_e f^2$

$P_H = K_H f$

$K_e (25)^2 + K_h (25) = 500 \text{ w}$

$K_e (50)^2 + K_h (50) = 1400 \text{ w}$

$K_e = 0.32 \qquad K_h = 12$

@ 25 Hz

$P_e = 0.32 \times 25^2 = 200 \text{ w}$

$P_h = 12 \times 25 = 300 \text{ w}$

@ 50 Hz:

$P_e = 800 \text{ w}$

$P_h = 600 \text{ w}$

P-20

energy loss per loop = 300 * volume

energy loss per hour = loss per loop * (23 x 33 x 106)

$= 300 * \frac{m}{\rho} * 90000$

$= 300 * \frac{12}{7.5 \times 10^3} * 90000$

loss per hour = 43200 J

Scanned by CamScanner

Date: NO:

P-21

$A_g = 2.5\ cm^2 \quad l_g = 0.4\ cm \quad B_g = 0.8\ T$

$H_m l_m + H_g l_g = 0$

$l_m = \frac{H_g l_g}{H_m}$

$H_g = \frac{B_g}{\mu_0} = 6366 l9.7 \quad AT/m$

From B-H curve: max $H_m B_m$ at the operating point:

$B = 1\ T \quad H = -35\ KAT/m$

$\boxed{l_m = 7.27\ cm}$

$B_m A_m = B_g A_g$

$A_m = \frac{B_g A_g}{B_m} = \frac{0.8 \times 2.5}{1} = \boxed{2\ cm^2}$

P-22 $l_g = 0.4\ cm$

$l_m = 2.4\ cm \quad A_m = 4\ cm^2$

$H_m l_m + H_g l_g = 0$

$H_m l_m + \frac{B_g}{\mu_0} l_g = 0 \quad \rightarrow (1)$

$B_g A_g = B_m A_m$

$B_g = \frac{B_m A_m}{A_g}$

Scanned by CamScanner

Date:

$$H_m L_m = -\frac{L_g}{\mu_0}\frac{B_m A_m}{A_g}$$

$$\boxed{B_m = -6\mu_0 H_m}$$ solve with B-H curve

$B_m = 0.33\,T \qquad H_m = -43.76\ kAT/m$

$B_g = B_m \qquad (A_g = A_m)$

---

p_23 For minimum volume → max $H_m B_m$

the operating point at:

$B_m = 1\,T \qquad H_m = -40\ kAT/m$

$H_m L_m + H_g L_g = 0$

$-40\times10^{3} L_m + \frac{0.8}{\mu_0} \times 4\times10^{-3} = 0$

$$\boxed{L_m = 6.366\ cm}$$

$B_g A_g = B_m A_m$

$$\boxed{A_m = \frac{0.8\times2.5}{1} = 2\ cm^2}$$

Scanned by CamScanner

Date: no:

P-23 ①

$$H_m L_m + H_g L_g = 0$$

$$H_g = \frac{B_g}{\mu_0} = -\frac{H_m L_m}{L_g}$$

$$B_g = -\frac{\mu_0 H_m L_m}{L_g}$$

$$B_g A_g = B_m A_m$$

$$B_g = \frac{B_m A_m}{A_g}$$

$$B_g^2 = -\frac{\mu_0 H_m L_m B_m A_m}{L_g A_g} = -\frac{\mu_0 H_m B_m \forall_m}{\forall_g}$$

$$\forall_m = -\frac{B_g^2 \forall_g}{\mu_0 H_m B_m}$$

from B-H curve

$B_m = 1 \quad H_m = -35 \text{ KAT/m}$

$$V_m = \frac{(0.7)^2 \times 3 \times 0.4}{\mu_0 \times 1 \times 35 \times 10^3} = 13.369 \text{ cm}^3$$

Scanned by CamScanner